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**COMPARISON OF FINITE SOURCE AND
PLANE WAVE SCATTERING FROM
CORRUGATED SURFACES**

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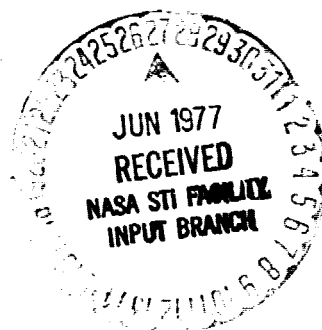
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D. M. Le VINE

APRIL 1977



GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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ABSTRACT

The choice of a plane wave to represent incident radiation in the analysis of scatter from corrugated surfaces is examined. This is done by comparing the physical optics solution obtained for the scattered fields due to an incident plane wave with the solution obtained when the incident radiation is produced by a source of finite size and finite distance from the surface. The two solutions are equivalent if the observer is in the far field of the scatterer and the distance from observer to scatterer is large compared to the radius of curvature at the scatter points, conditions not easily satisfied with extended scatterers such as rough surfaces. In general, the two solutions have essential differences such as in the location of the scatter points and the dependence of the scattered fields on the surface properties. The implication of these differences to the definition of a meaningful radar cross section is examined. It is shown that the radar cross section defined from incident plane waves is meaningful for the case of finite sources and extended scatterers if the far field conditions are met or, if the scatter is incoherent and the surface statistically homogeneous, whenever the observer is far from the surface compared to its radius of curvature at the scatter points.

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COMPARISON OF FINITE SOURCE AND PLANE WAVE SCATTERING FROM CORRUGATED SURFACES

The choice of a plane wave to represent incident radiation is a common and useful approximation in scattering problems. On the other hand there are many situations in active (radar) sensing of the earth from airborne and space platforms where the specific phase structure of the incident radiation can be important. Also, in cases of extended scatterers there is no a priori guarantee that radar cross section as derived from analysis of plane waves is applicable for predicting scattered power. It is the objective of this article to compare the scattered fields obtained by assuming an incident plane wave with those obtained by assuming a finite source for the case of scatter from stochastic surfaces. Some obvious differences are manifest, such as amplitude decrease due to wave spreading, present in the case of radiation from a finite source but not in the case of plane waves, and differences in the temporal history of the received signal which are a consequence of the plane phase structure of the incident radiation with plane waves and the curved structure with finite sources. More subtle differences involving the dependence of the solutions on the parameters of the surface also appear, and may have consequences in interpretation of scattering cross section.

The specific example to be considered here is scattering from irregular (i. e. , stochastic) surfaces with the restriction to perfectly conducting boundaries

and two dimensions. This is an idealized model relevant in first order for scattering from ocean surfaces and relevant to some extent for scattering from plowed fields and perhaps rough snow and ice. The restriction to two dimensions has been chosen for the obvious simplification in treating the vector problem. A physical optics solution (Kirchhoff approximation plus asymptotic evaluation of the Helmholtz integral) is obtained, and is a reasonable approximation for microwave radar scattering near nadir, at least for ocean surfaces (Bass, et al. , 1968; Barrick, 1968a-b). The object of the analysis is to compare the fields scattered back to an arbitrarily located observer when, in the first case, the incident signal is a plane wave, $\vec{E}(r, \nu) = \vec{E}_0 e^{jk \cdot \vec{r}}$, and in the second case in which the finite nature of the radar antenna is taken into account. The geometry is illustrated in Figure 1.

In the following sections the scattered fields will be computed for the two cases and a comparison made. Then, a radar cross section applicable to extended scatterers will be defined from the radar equation, and the solutions for the scattered fields will be used to identify conditions under which this definition of cross section corresponds to the conventional definition obtained from incident plane waves.

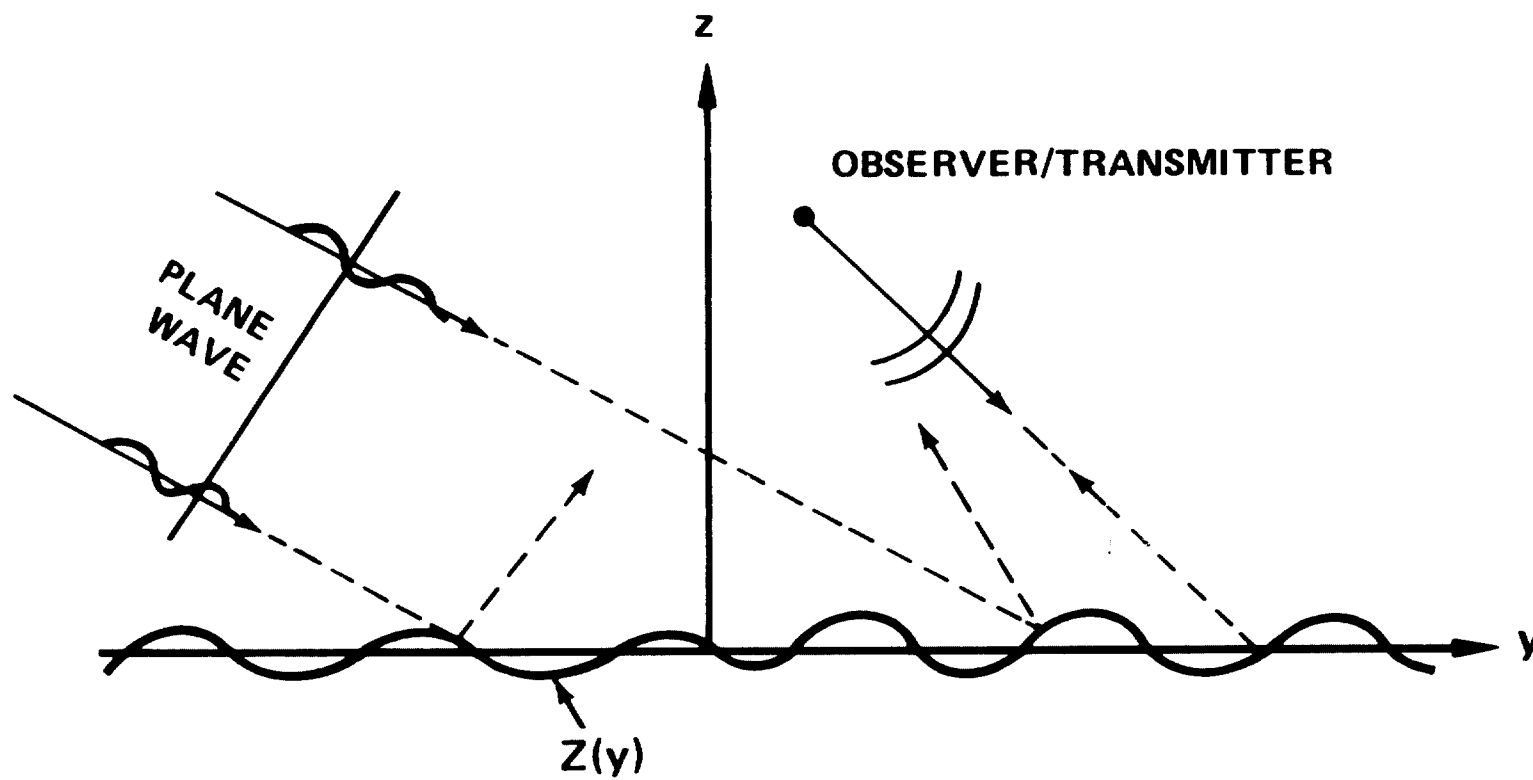


Figure 1. Scattering Geometry

SCATTERED FIELDS

Both solutions for the scattered electric field, $\bar{e}_s(r, \nu)$, can be written as follows in the case of perfectly conducting boundaries:

$$\bar{e}_s(\bar{r}, \nu) = - \oint\!\!\!\oint_{\text{surface}} \frac{\partial \bar{e}}{\partial n} g(\bar{r}/\bar{r}') ds \quad (1)$$

where $g(\bar{r}/\bar{r}')$ is the two-dimensional Green's function, $j/4 H_0^{(1)}(k|\bar{r} - \bar{r}'|)$, and $\hat{n}$ is a unit vector normal to the surface, $Z(y)$. Assuming perpendicular polarization ($\bar{E}_0 = E_0 \hat{x}$), the Kirchhoff approximation yields the following result in the case of plane waves:

$$\frac{\partial \bar{e}}{\partial n} = 2j(\bar{k} \cdot \hat{n}) \tilde{j}(\nu) E_0 \hat{x} e^{j\bar{k} \cdot \bar{r}} \quad (2)$$

where $\tilde{j}(\nu)$ is the Fourier transform (temporal) of the incident pulse. In the case of a finite source, assuming that the source can be represented in terms of an equivalent current distribution, $\hat{x} j(\nu) J(x, y)$, the Kirchhoff approximation yields the following result (Appendix A):

$$\frac{\partial \bar{e}}{\partial n} = \hat{x} \sqrt{\mu/\epsilon} j(\nu) \int_{\text{source}} \frac{k^2 z(y')}{2R} J(y', z') H_0^{(1)'}(kR) d\bar{r}' \quad (3)$$

where $R = \sqrt{(y-y')^2 + Z^2(y')}$ and $H_0^{(1)'}(kR)$ is the derivative with respect to kR of the Hankel function.

The scattered fields are obtained by substituting Equation 2 or 3, as appropriate, into Equation 1. Performing the integration in the limit that $kR \rightarrow \infty$ by means of a saddle point approximation (Copson, 1971), yields the following result for backscatter in the case of plane waves:

$$\begin{aligned} \bar{e}_s(\bar{r}, \nu) = & -j E_0 \hat{x} \sum_{\text{all } y_n} \frac{\cos(\theta - \alpha)}{\cos(\theta + \alpha)} e^{jk\Phi(y_n)} \cdot \\ & \cdot \left[1 - \frac{2 \cos \theta}{\cos \alpha \cos^2(\theta + \alpha)} \cdot \frac{R(y_n)}{R_c(y_n)} \right] \end{aligned} \quad (4)$$

where $\theta = \tan^{-1} \left[\left| \frac{kz}{ky} \right| \right]$ is the angle of incidence; $\alpha(y_n) = \tan^{-1} \left[\frac{\partial Z}{\partial y} \right]$ is the slope of the surface; $R(y_n)$ is the distance between the scatter point at y_n and the observer; $R_c(y_n)$ is the radius of curvature of the surface at the scatter point, and

$$\Phi(y_n) = y_n \sin \theta - Z(y_n) \cos \theta + R(y_n) \quad (5)$$

The scatter points, y_n , are determined by the requirement that $\left. \frac{\partial \Phi}{\partial y} \right|_{y_n} = 0$;

and for backscatter the requirement $\theta \leq \alpha$ pertains.

In order to obtain the equivalent expression in the case of a finite source, Equation 3 must first be evaluated. For this purpose, assume that the surface is far enough from the source to permit the fraunhofer approximation to be

made in Equation 3. Then, using an asymptotic form for $H_0^{(1)'}(kR)$ and using a binomial expansion for R one obtains (Appendix B):

$$\frac{\partial \bar{e}}{\partial n} = 1/2 \sqrt{\mu/\epsilon} j(\nu) k^2 \cos(\varphi - \alpha) \tilde{H}_0^{(1)'}(kR) F(y, \nu) \quad (6)$$

where R is the distance from the center of the source to the surface and $F(y, \nu)$, is the Fourier transform of $J(yz)$ evaluated at spatial frequencies, $\nu_y = \frac{\nu}{c} \sin \varphi$ and $\nu_z = -\frac{\nu}{c} \cos \varphi$ where

$$\varphi(y) = \cos^{-1} \left[\frac{H - Z(y)}{R} \right] \quad (7)$$

H is the height of the source above the mean surface (i. e., the ordinate of the source). The tilde ($\sim$) above $H_0^{(1)'}(kR)$ in Equation 6 denotes the asymptotic form for large kR , and the prime indicates the derivative with respect to R : $\tilde{H}_0^{(1)'}(kR) \triangleq jk \sqrt{z/kR} \exp [j(kR - \pi/4)]$. Substituting Equation 6 into Equation 1, and evaluating the integral in the limit of large kR , one obtains:

$$\bar{e}_s(\bar{r}, \nu) = \hat{x} \sqrt{\mu/\epsilon} j(\nu) \sqrt{k/16\pi} e^{-j\pi/4} \sum_{\text{all } y_n} \frac{F(y_n, \nu)}{\sqrt{R(y_n)}} e^{j2kR(y_n)} \cdot \left[1 - \frac{R(y_n)}{R_c(y_n)} \right]^{-1/2} \quad (8)$$

Equation 8 is the electric field scattered back to the finite source.

In order to compare Equations 4 and 8, it is first necessary to find an equivalent in the case of the finite source for the amplitude of the plane wave. For this purpose assume that the amplitude, E_0 , of the plane wave, is the magnitude in the fraunhofer limit of a cylindrical wave radiated by the finite source. Since the far field radiated by the finite source in the direction of incidence of the plane wave (i. e. , θ) is:

$$\begin{aligned}\bar{e}(\bar{r}, \nu) &= jkc \hat{x} A_x \\ &= -\hat{x} \sqrt{\mu/\epsilon} j(\nu) \sqrt{k/8\pi} e^{-j\pi/4} \frac{e^{jkR}}{\sqrt{R}} \int_{-\infty}^{\infty} J(y, z) e^{jk[y \sin \theta - z \cos \theta]} dy dz \\ &= -\hat{x} \sqrt{\mu/\epsilon} j(\nu) \sqrt{k/8\pi} e^{-j\pi/4} \frac{e^{jkR}}{\sqrt{R}} F(\theta, \nu)\end{aligned}\quad (9)$$

one obtains:

$$E_0 \tilde{j}(\nu) = -\sqrt{\mu/\epsilon} e^{-j\pi/4} \sqrt{k/8\pi} j(\nu) F(\theta, \nu) \quad (10)$$

Using this definition, one obtains the following formulas for the scattered fields:

a) Plane wave

$$\bar{e}_s(\bar{r}, \nu) = -E_0 \tilde{j}(\nu) \hat{x} \sum_{\text{all } y_n} \frac{\cos(\theta - \alpha)}{\cos(\theta + \alpha)} e^{jk\Phi(y_n)} \left[1 - \frac{2 \cos \theta}{\cos \alpha \cos^2(\theta + \alpha)} \cdot \frac{R(y_n)}{R_c(y_n)} \right]^{-1/2} \quad (11a)$$

b) Finite source

$$\bar{e}_s(\bar{r}, \nu) = - \tilde{E}_0 \tilde{j}(\nu) \hat{x} \sum_{\text{all } y_n} \frac{\tilde{F}(y_n, \nu)}{\sqrt{2 R(y_n)}} e^{j_2 k R(y_n)} \left[1 - \frac{R(y_n)}{R_c(y_n)} \right]^{-1/2} \quad (11b)$$

where $\tilde{F}(y_n, \nu) = F(y_n, \nu)/F(\theta, \nu)$. If $F(\theta, \nu)$ is the maximum value of $F(y_n, \nu)$ then $\tilde{F}(y_n, \nu)$ is the relative field pattern for the source (Collin and Zucker, 1969). Also the following definitions have been made assuming the observer to be at $(0, H)$:

$$\Phi(y_n) = R(y_n) + y_n \sin \theta - Z(y_n) \cos \theta \quad (12a)$$

$$R(y_n) = \sqrt{[H - Z(y_n)]^2 + y_n^2} \quad (12b)$$

$$\tilde{E}_0 \tilde{j}(\nu) = - \sqrt{\mu/\epsilon} e^{-j \pi/4} \sqrt{k/8\pi} j(\nu) F(\theta, \nu) \quad (12c)$$

COMPARISON

In comparing Equations 11a and 11b it is important to keep in mind that Equation 11a represents the scattered fields when an incident plane wave (source at infinity) scatters to an observer at some finite distance from the surface; In contrast, Equation 11b represents the fields in the case of backscatter from a source of finite size and at finite distance from the surface. In the former case, plane phase fronts scatter (in the physical optics limit) as cylindrical waves back to the source whereas in the latter case, incident cylindrical waves scatter as cylindrical waves back to the source. This difference accounts for the additional distance factor, $\sqrt{R(y_n)}$, in the denominator in Equation 11b: it represents cylindrical spreading of waves on transmission from the finite source.

Other amplitude differences as well as differences in phase and location of the scattering points also occur. For example, consider the location of the scatter points, y_n . The physical optics solution requires that in both cases these be "specular" points -- angle of incidence equals angle of reflection relative to the local surface normal -- but the geometrical possibilities are much different in the two cases. In the case of an incident plane wave all incident rays have the same direction whereas for the finite source they are all radial vectors emanating from the source. As a consequence, in the case of the finite source, backscatter requires incident and reflected rays to

coincide and to be perpendicular to the surface at the scatter point. In contrast with an incident plane wave all manner of variations are possible including what should rightly be called forward scatter. (See Figure 1.) As a result the distances, $\Phi(y_n)$ and $R(y_n)$, are not the same in the two solutions.

Only if one restricts the size of the illuminated surface will $\Phi(y_n)$, $R(y_n)$ and the location of the scatter points, y_n , be comparable in the two cases. For example, if one were to require that the observer be in the "far field" of the illuminated surface, then expanding $R(y)$ in a binomial series about the distance R_0 from observer to center of the illuminated footprint, one obtains:

$$R(y) \simeq R_0 + y \sin \theta - Z(y) \cos \theta \quad (13a)$$

and consequently

$$\Phi(y) \triangleq R(y) + y \sin \theta - z(y) \cos \theta \quad (13b)$$

$$\simeq 2R(y) - R_0$$

That is, except for the constant phase factor, kR_0 , which is arbitrary for the plane wave, the two phase factors are essentially equal when the scattering surface is small. Consequently, in this case the location of the scatter points will be the same for either type of incident variations.

But even with the far field restriction, the amplitude terms in Equations 11a and 11b are different. That is, even with $\theta = \alpha$ in Equation 11a and assuming

the scatter points are the same, there is a fundamental difference in the radicals. This is again a manifestation of the different phase structure -- plane versus cylindrical -- incident at the scatter point in the two cases. Only if the distance from surface to observer is much greater than the radius of curvature at the scatter point (i. e. , $R(y_n) \gg R_c(y_n)$) do the radicals become equal. This is because in the physical optics solution the incident signal scatters from an equivalent mirror at the surface with focal length, $R_c(y_n)$, and the phase distribution along this infinitesimal mirror is functionally different in the two cases treated here.

IMPLICATIONS FOR RADAR CROSS SECTION

The restrictions mentioned above are sufficient to insure that Equations 11a and 11b are essentially term-wise equivalent (with appropriate consideration given to antenna gain and range effects). That is, if the observer is in the far field of the scatterer and sufficiently removed that the distance to the scatterer is much greater than the radius of curvature at the scattering points, then Equations 11a and 11b differ only in factors due to the radiation pattern of the antenna and cylindrical spreading of the incident radiation which are present in the case of the finite source.

The interesting case occurs for extended scatterers such as irregular surfaces where one can be far from the surface but receive energy scattered from a wide range of incidence angles. Such situations arise, for example, in monitoring ocean surfaces either with an altimeter where the pulse first intersects the surface at nadir and then at increasing incidence angles as a function of time (Barrick, 1972), or in such off nadir sensing configurations as is employed with the Short Pulse Radar where a long narrow antenna beam directs the pulse along the surface from an initial intersection near nadir to an eventual intersection at 30° or more (Tomiyasu, 1971; Le Vine et al., 1975).

In case of extended scatterers one might imagine computing the scattered power as a sum over the power scattered by rays incident at the various angles allowed

by the scattering geometry. For example, based on the radar equation, one might write (in two-dimensions) the following form for the received power:

$$P_R(\nu) = \int_{\text{surface}} \frac{4}{k} \frac{P_t(\nu) G_t(\theta) G_R(\theta)}{(2 \pi R)^2} \sigma^\circ(\theta) ds \quad (14)$$

where $\sigma^\circ(\theta)$ is a cross section per unit surface (length in this two-dimensional example). A three dimensional equivalent to this equation has been used to analyse the performance of a radar altimeter (e. g. , Harger, 1972) and has been used traditionally in interpretation of radar scatter from the ocean (Kerr, 1951). (See Appendix C for definition of terms.)

Equation 14 can be regarded as a definition of $\sigma^\circ(\theta)$: That is, one determines $\sigma^\circ(\theta)$ by formally computing the received power using solutions for the scattered fields (such as Equation 11b) and putting the results in the form of Equation 14. In two important cases this procedure leads to the same cross section as obtained with the definition based on incident plane waves (Kerr, 1951; Skolnik, 1970). These cases are: 1) When the observer is in the far field of the scatterer and far away compared to the radius of curvature at the scatter points; and 2) When the scatter is incoherent, the surface is statistically homogeneous (i. e. , spatially stationary) and the observer is far from the surface compared to the radius of curvature at the scatter points.

The equivalence in the first case is a consequence of the discussion in the previous section. In this case the scatterer can be regarded as essentially a single point and the conditions of the previous section are adequate to insure that the two solutions, Equations 11a and 11b are identical.

In order to demonstrate the equivalence in the second case expressions are necessary for the radar cross section from Equation 14 and as defined by the assumption of incident plane waves. The expression for the radar cross section as obtained from plane waves is obtained by a straight forward substitution of the plane wave solution, Equation 11a, into the definition of cross section (Kerr, 1951; Skolnik, 1970). With minor generalizations to specialize the results to two dimensions, pulses and stochastic scatters, one obtains:

$$\langle \sigma_p^\circ(\theta) \rangle = \frac{1}{L} \lim_{\rho \rightarrow \infty} 2\pi\rho \left\langle \left| \frac{\bar{e}_s(\bar{r}, \nu) * \bar{e}_s(\bar{r}, \nu)}{E_o \tilde{j}(\nu) * E_o \tilde{j}(\nu)} \right| \right\rangle \quad (15)$$

where the brackets $\langle \rangle$ denote an ensemble (statistical) average and the asterisk-dot ($*$) denotes a convolution of the pairs in a scalar (dot) product. (The convolution is the frequency domain equivalent of a product in the time domain and reduces to a simple product in the case of harmonic time dependence and a complex analytic representation of the fields.) L is the length of the illuminated surface. The scattered field, $\bar{e}_s(\bar{r}, \nu)$, is given by Equation 11a. In the special case $R(y_n) \gg R_c(y_n)$ and incoherent scatterers, Equation 15 becomes:

$$\langle \sigma_p^\circ(\theta) \rangle = \frac{\pi}{L} \sum_{\text{all } y_n} \langle |R_c(y_n)| e^{jk[\varphi(y_n) - \varphi_0]} \rangle \quad (16)$$

where φ_0 is an arbitrary reference phase. When the surface is homogeneous, this solution reduces to:

$$\langle \sigma_p^\circ(\theta) \rangle = \pi \eta \langle |R_c(y_n)| e^{jk[\varphi(y_n) - \varphi_0]} \rangle \quad (17)$$

where η is the number of scatterers per unit length.

The equivalent expression for distributed scatterers is obtained by computing scattered power from Equation 11b and putting the result in to the form of Equation 14. The received power is obtained from Equation 11a as follows:

$$\langle P_r(\nu) \rangle = \sqrt{\epsilon/\mu} \langle \left[\bar{e}_s(\bar{r}, \nu) \sqrt{\frac{G_R(\bar{r}, \nu)}{k/4}} \right] \cdot \left[\bar{e}_s(\bar{r}, \nu) \sqrt{\frac{G_R(\bar{r}, \nu)}{k/4}} \right] \rangle \quad (18)$$

where $\frac{G_R(\bar{r}, \nu)}{k/4}$ is the equivalent "area" of the receiving antenna and $G_R(\bar{r}, \nu)$

its gain (Friis and Lewis, 1947). Assuming that $R(y_n) \gg R_c(y_n)$ and the scatter is incoherent, one obtains (Appendix C):

$$\langle P_r(\nu) \rangle = \langle \sum_{\text{all } y_n} \frac{4}{k} \frac{P_t(\nu) G_R(y_n) G_t(y_n)}{[2 \pi R(y_n)]^2} \left[\pi |R_c(y_n)| e^{j2k[R(y_n) - R_0]} \right] \rangle \quad (19)$$

Assuming homogeneous surface statistics and neglecting the small dependence of the antenna parameters (and distance) on the surface heights, this

expression can be written:

$$\langle P_r(\nu) \rangle = \int_{\text{surface}} \frac{4}{k} \frac{P_t(\nu) [G_R G_t]^2}{(2\pi R)^2} \left[\pi \eta \langle |R_c(y_n)| e^{j2k [R(y_n) - R_o]} \rangle \right] dy \quad (20)$$

Comparison with Equation 14 yields the following form for $\langle \sigma^\circ(\theta) \rangle$:

$$\langle \sigma^\circ(\theta) \rangle = \pi \eta \langle |R_c(y_n)| e^{j2k [R(y_n) - R_o]} \rangle \quad (21)$$

which is the same as Equation 17 and is the two dimensional equivalent of results which have appeared in the literature on scattering from ocean surfaces (Barrick, 1968; Kodis, 1966).

The important point to be made here is not so much the equivalence of Equation 17 and 21, but rather that the equivalence is the consequence of several assumptions (far field, $R(y_n) \gg R_c(y_n)$, incoherence and homogeneity). These assumptions are, of course, not universally true. It was shown above that the expression for the scattered fields, assuming incident plane waves and waves incident from a finite source, are not identical unless the observer is in the far field of the surface and much further away than any significant radius of curvature on the surface. It follows, that the plane wave expression for cross section, $\sigma^\circ(\theta)$, is not always applicable for use in Equation 14.

CONCLUSIONS

It has been shown that in order for the scattered fields obtained using a finite source and incident plane waves to agree, two conditions must be satisfied: 1) The scatter must be of finite extent with the observer in the far field of the scatterer; and 2) The distance from scatter points to the observer must be much greater than the radius of curvature of the surface at the scatter point. These conditions guarantee that the solutions differ only in the obvious spreading due to the cylindrical nature of the incident energy in the case of a finite source. If the preceding two conditions are met, and one properly keeps track of the spreading, then a radar cross section can unambiguously be defined using the definition based on plane waves. Otherwise, the equivalence of radar cross section as defined for incident plane waves and from the radar equation only pertains in special cases. One such special case occurs for incoherent scatter from statistically homogeneous surfaces if the observer is far from the surface as compared to the radius of curvature of the surface at the scatter points.

APPENDIX A

FIELDS DUE TO A SOURCE ABOVE
A PERFECTLY CONDUCTING PLANE

Consider the two dimensional problem of finding the electric field due to a current source which is above a perfectly conducting plane and directed parallel to the surface. In the case of an x-directed current, the x-component of magnetic potential is adequate to describe the fields, and the electromagnetic fields are given by:

$$\vec{e}(\vec{r}, \nu) = j\omega A_x(\vec{r}, \nu) \hat{x} \quad (A1)$$

$$\vec{h}(\vec{r}, \nu) = \frac{1}{\mu} \vec{\nabla} \times [A_x(\vec{r}, \nu) \hat{x}] \quad (A2)$$

The magnetic vector potential $A_x(\vec{r}, \nu)$ can be expressed in terms of the two dimensional Green's function, $g_x(\vec{r}/\vec{r}')$, as follows:

$$A_x(\vec{r}, \nu) = \mu \int J_x(\vec{r}', \nu) g_x(\vec{r}/\vec{r}') d\vec{r}' \quad (A3)$$

The Green's function can be represented as a sum of a free space (no boundaries) Green's function plus a scatter term due to the boundaries:

$$g_x(\vec{r}/\vec{r}') = g_x^0(\vec{r}/\vec{r}') + g_x^s(\vec{r}/\vec{r}') \quad (A4)$$

and the scattered term can be obtained by finding the plane wave representation of $g_x^0(\vec{r}/\vec{r}')$ and then constructing $g_x^s(\vec{r}/\vec{r}')$ out of the same superposition but using the scattered (reflected) plane waves (Clemmow, 1956). Doing so,

one obtains:

$$g_x^0(\bar{r}/\bar{r}') = \frac{j}{4} H_0^{(1)}(k\gamma) \quad (A5)$$

$$g_x^s(\bar{r}/\bar{r}') = -\frac{j}{4} H_0^{(1)}(k\tilde{\gamma}) \quad (A6)$$

where for a source at $(y'; z')$ (Figure 2) one has:

$$\gamma = \sqrt{(y - y')^2 + (z - z')^2}$$

$$\tilde{\gamma} = \sqrt{(y - y')^2 + (z + z')^2}$$

The scatter component is just the free space Green's function due to a point source at z' units below the surface with current in the negative x -direction. Using Equations A5 and A6, one obtains the following result for the x -component of the magnetic vector potential:

$$A_x(\bar{r}, \nu) = \frac{j\mu}{4} \int J_x(\bar{r}', \nu) \left[H_0^{(1)}(k\gamma) - H_0^{(1)}(k\tilde{\gamma}) \right] d\bar{r}' \quad (A7)$$

Substituting this expression for $A_x(\bar{r}, \nu)$ into Equation A1, one obtains

$$e_x(\bar{r}, \nu) = -\sqrt{\mu/\epsilon} k/4 \int J_x(\bar{r}', \nu) \left[H_0^{(1)}(k\gamma) - H_0^{(1)}(k\tilde{\gamma}) \right] d\bar{r}' \quad (A8)$$

It is trivial to check that this expression is zero in on the surface (i. e. , at $\gamma = \tilde{\gamma}$). The derivative of $e_x(\bar{r}, \nu)$ normal to the surface is also easily computed:

$$\frac{\partial e_x(\bar{r}, \nu)}{\partial z} = -\sqrt{\mu/\epsilon} k^2/4 \int J_x(\bar{r}', \nu) \left\{ \frac{z - z'}{\gamma} H_0^{(1)'}(k\gamma) - \frac{z + z'}{\tilde{\gamma}} H_0^{(1)'}(k\tilde{\gamma}) \right\} d\bar{r}' \quad (A9)$$

and when evaluated on the surface yields:

$$\left. \frac{\partial e_x(\bar{r}, \nu)}{\partial z} \right|_{z=0} = \sqrt{\frac{\mu}{\epsilon}} \int \frac{k^2 z'}{2\gamma} J_x(\bar{r}', \nu) H_0^{(1)'}(k\gamma) d\bar{r}' \quad (\text{A10})$$

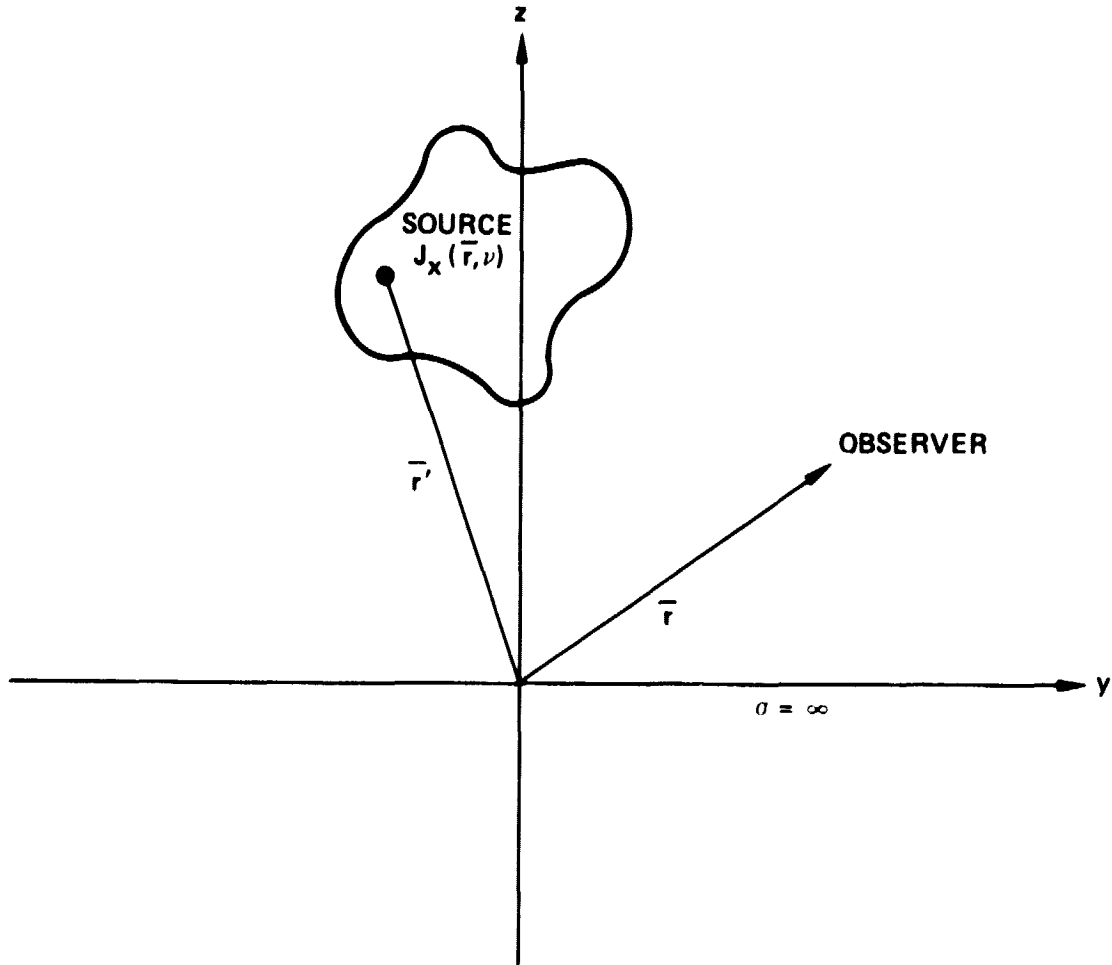


Figure 2. Geometry for the Calculation of the Fields Due to a Source Above a Perfectly Conducting Plane

APPENDIX B

INTEGRATION OVER THE SOURCE COORDINATES

It was shown in Appendix A that in the case of a line source, $J_x(\bar{r}, \nu)$, above a perfectly conducting plane, the derivative of electric field intensity normal to the surface and evaluated on the surface ($z = 0$) is:

$$\left. \frac{\partial e_x(\bar{r}, \nu)}{\partial z} \right|_{z=0} = \sqrt{\frac{\mu}{\epsilon}} \int \frac{k^2 z'}{2\gamma(\bar{r}')} J_x(\bar{r}', \nu) H_0^{(1)'}(k\gamma) d\bar{r}' \quad (B1)$$

where

$$\gamma(\bar{r}') = \sqrt{(y - y')^2 + (z')^2}.$$

The coordinates in Equation B1 refer to the configuration shown in Figure 2; however, because this analysis is to be applied to a plane tangent to the stochastic surface, $Z(y)$, at many different points, it is necessary to express all coordinates in terms of a reference coordinate system (y, z) as shown in Figure 3. The coordinates in which Equation B1 is presently expressed are the primed system in Figure 3. The origin of the primed system has been chosen so that the z' axis passes through the center of mass of the source because it is convenient to do the integration over the source coordinates in a center of mass system. Letting the center of mass system be (η, ξ) with axes parallel to those of the reference system, Equation B1 becomes:

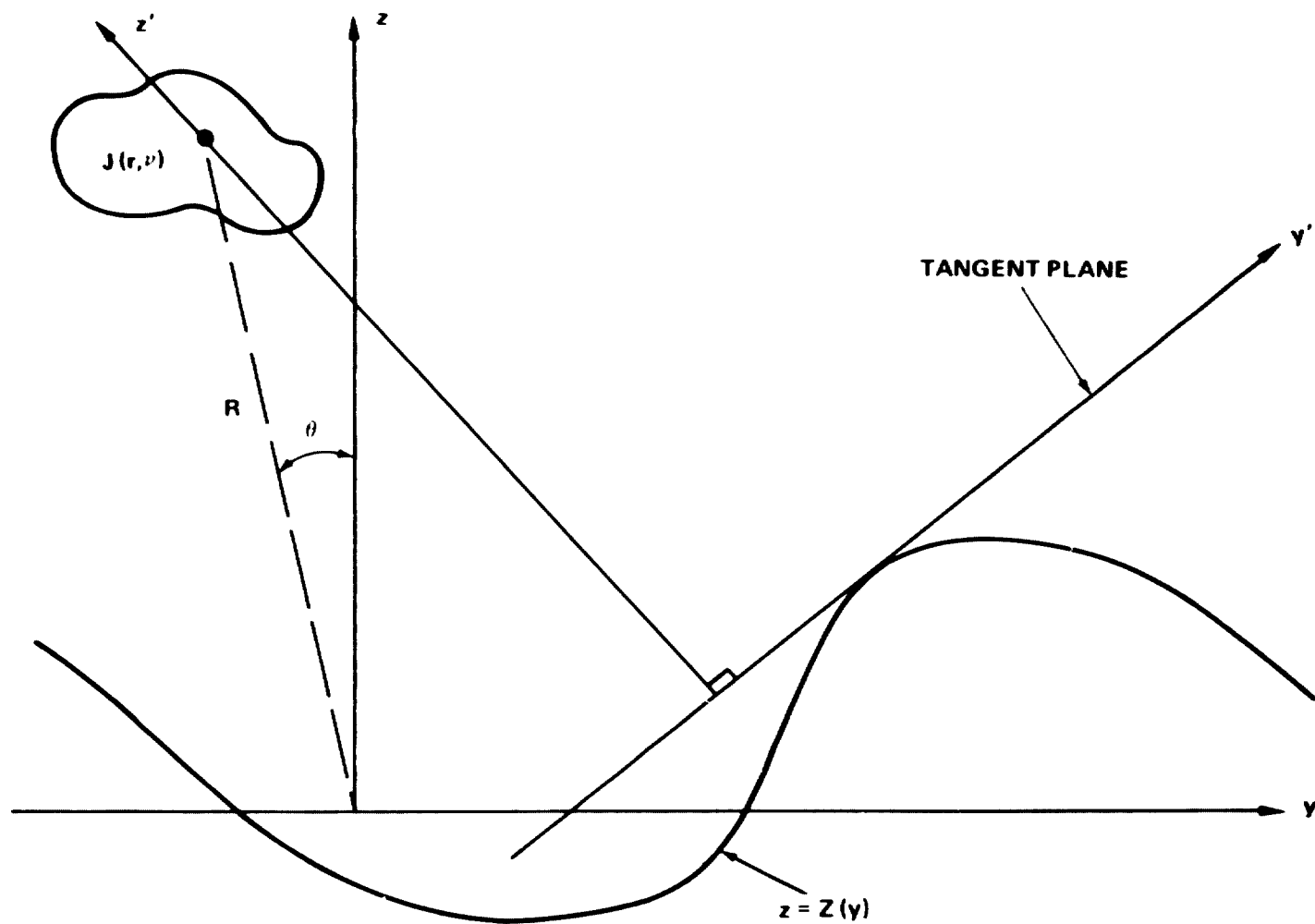


Figure 3. Local Coordinate System for Calculating the Fields on the Surface (Kirchhoff Approximation)

$$\left. \frac{\partial e_x}{\partial z} \right|_{z=0} = \sqrt{\frac{\mu}{\epsilon}} \int \frac{k^2 [Z'(\xi, \eta)]}{2\gamma(\eta, \xi)} J(\eta, \xi, \nu) H_0^{(1)'}(k\gamma) d\eta d\xi \quad (B2)$$

where:

$$\gamma(\eta, \xi) = \sqrt{[\rho(y) \cos \varphi + \xi]^2 + [\rho(y) \sin \varphi - \eta]^2}$$

$$Z'(\eta, \xi) = \rho(y) \cos(\varphi - \alpha) + [\xi \cos \alpha - \eta \sin \alpha]$$

$$\rho(y) = \sqrt{[R \cos \theta - Z(y)]^2 + [R \sin \theta + y]^2}$$

$$\phi(y) = \tan^{-1} \left[\frac{R \cos \theta - Z(y)}{R \sin \theta + y} \right]$$

Although this integration can not be done in general, it can be approximated in the important special case in which the source is in the far field of the surface in terms of the distance, $\rho(y)$, between its center of mass and the point at which the plane is tangent to $z(y)$. In this case, employing an asymptotic form for the Hankel function, Equation B2 becomes:

$$\left. \frac{\partial e_x}{\partial z} \right|_{z=0} \cong \sqrt{\frac{\mu}{\epsilon}} \frac{k^2}{2} H_0^{(1)'}(k\rho) \iint \cos(\varphi - \alpha) \cdot J(\xi, \eta, \nu) e^{j2\pi[a(y)\eta + b(y)\xi]} d\eta d\xi \quad (B3)$$

where

$$a(y) = -\frac{\nu}{c} \sin[\varphi(y)]$$

$$b(y) = \frac{\nu}{c} \cos[\varphi(y)]$$

The limits integration may be formally extended to include the entire η, ξ space, and then one obtains:

$$\left. \frac{\partial e_x}{\partial z} \right|_{z=0} = \sqrt{\mu/\epsilon} \frac{k^2}{2} H_0^{(1)'}(k\rho) \cos(\varphi - \alpha) \tilde{G}(y, \nu) \quad (B4)$$

where $\tilde{G}(y, \nu)$ is a Fourier transform of the spatial coordinates of the source, $J(\eta, \xi, \nu)$, evaluated at frequencies $\nu_\eta = a(y)$ and $\nu_\xi = b(y)$. Whenever the spatial and temporal dependence in the source are separable [i. e., $J(r, \nu) = J_r(r) j(\nu)$] one obtains:

$$G(y, \nu) = j(\nu) \mathfrak{F} [J_r(\eta, \xi)] \left| \begin{array}{l} \nu_\eta = a(y) \\ \nu_\xi = b(y) \end{array} \right. \quad (B5)$$

where $\mathfrak{F}$ denotes a Fourier transform.

An important special case is that in which $J_r(\eta, \xi)$ is a delta function (i. e., one has a point source). In this case the integrations are easily evaluated to yield:

$$G(y, \nu) = j(\nu) \quad (B6)$$

It is convenient in employing these results to separate explicitly the spectrum of the current waveform, $j(\nu)$, and the spatial character of the source. Thus, let $F(y, \nu)$ be defined by:

$$G(y, \nu) = j(\nu) F(y, \nu) \quad (B7)$$

It is the factor, $F(y, \nu)$, which is employed in the text.

The gain, $G_t(\varphi)$, of the transmitting antenna is defined to be the power radiated per unit surface in the direction, φ , divided by the total power radiated per unit surface (Collin and Zucker, 1969). That is:

$$G_t(\varphi) = \frac{F^2(\varphi)}{\frac{1}{2\pi} \int_0^{2\pi} F^2(\varphi) d\varphi} \quad (C3)$$

The power received by an antenna is equal to the incident power density weighted by the effective area of the receiving antenna (which constitutes a definition of "effective area"). This effective area can be related to antenna gain by means of the reciprocity theorem. In the case of two dimensions the effective area of an antenna with gain, $G_r(\bar{r}, \nu)$, is (Friis and Lewis, 1947):

$$A_r(\bar{r}, \nu) = (4/k) G_r(\bar{r}, \nu) \quad (C4)$$

The received power is given in terms of the effective area, $A_r(\bar{r}, \nu)$, and the scattered electric field, $\bar{e}_s(\bar{r}, \nu)$, by:

$$P_r(\nu) = \sqrt{\epsilon/\mu} \left[\bar{e}_s(\bar{r}, \nu) \sqrt{A_r(\bar{r}, \nu)} \right] \cdot \left[\bar{e}_s(\bar{r}, \nu) \sqrt{A_r(\bar{r}, \nu)} \right] \quad (C5)$$

The form required in the text for received power scattered from the surface is obtained by substituting Equation 11b for the scattered fields into Equation C5 above. Using the preceding definitions Equation 11b takes the form:

$$\bar{e}_s(\bar{r}, \nu) = \hat{x} \sqrt{\mu/\epsilon} \sum_{\text{all } y_n} \frac{I_o(\nu)}{\sqrt{2 R(y_n)}} \left[\frac{G_t(\varphi)}{2\pi} \int_0^{2\pi} F^2(\varphi) d\varphi \right] \left[1 - \frac{R(y_n)}{R_c(y_n)} \right]^{\frac{1}{2}} e^{j2k [R(y_n) - R_c(y_n)]} \quad (C6)$$

where

$$I_o(\nu) = j(\nu) \sqrt{k/\delta\pi} \exp \left\{ j \left[2k R_o(y_n) - \pi/4 \right] \right\} \quad (C7)$$

$R_o(y_n)$ is the distance from the observer to the mean surface at the scatter point, y_n . It has been introduced here to reference the time of reception of the scattered pulse to the round trip propagation time between source and surface. In this way, fluctuations about this mean time due to surface irregularities are included explicitly in the factor: $\exp \left\{ j2k \left[R(y_n) - R_o(y_n) \right] \right\}$.

Now assuming that $R(y_n) \gg R_c(y_n)$ and that the scatter is incoherent, one obtains the following expression for the mean received power:

$$\begin{aligned} \langle P_r(\nu) \rangle &= \sqrt{\mu/\epsilon} \sum_{\text{all } y_n} 4/k \langle \left[I_o(\nu) * I_o(\nu) \int_0^{2\pi} F^2(\varphi) d\varphi \right] \\ &\quad \frac{G_t G_R}{(2\pi R)^2} \left[\pi |R_c(y_n)| e^{j2k \left[R(y_n) - R_o(y_n) \right]} \right] \rangle \\ &= \sum_{\text{all } y_n} 4/k \left\langle \left[\frac{P_t G_t G_R}{(2\pi R)^2} \right] \pi |R_c(y_n)| e^{j2k \left[R(y_n) - R_o(y_n) \right]} \right\rangle \end{aligned} \quad (C8)$$

which is the form used in the text.

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